Diffraction effects in microlensing: analytical results on the radiation power spectrum of a Gaussian source

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Abstract

We consider wave optics effects in the case of microlensing of an extended Gaussian source by a point mass under standard assumptions about the incoherent emission of the source elements. We obtain analytical expressions for the power spectrum of the microlensed radiation, which are convenient to obtain asymptotic formulae in the case of a large source. If the source centre, the lens and the observer are on a straight line, the power spectrum is presented in a closed form in terms of the hypergeometric function. In case of a general location of the lens and the source the expression for the power spectrum is presented in a form of a series. In the high frequency limit the results lead to usual expressions of the geometric optics. Approximate expressions for the power spectrum in case of a large source and high frequencies are obtained.

1 Introduction

Gravitational lens systems (GLS) have an important role for investigations of mass distributions in the Universe; in particular, the gravitational microlensing is a powerful tool in probing planetary and solar-mass objects

in the Milky Way (e.g. [20, 18, 22]). Most of the GLS studies is devoted to observable effects that can be described within geometric optics approximation ([15, 8]). The effects of the physical optics, however, could give an independent information about the lensing objects. Also, a detection of wavelength-dependent effects in GLS can open the way to entirely new tests of the electrodynamics in curved space-time. It has been repeatedly pointed out that, though for known GLS the wave effects are small, they may have an observational interest in the near future. Different theoretical aspects of the physical optics effects in GLS have been studied elsewhere (see a bibliography in monographs [2, 15] and in papers [9, 11, 12, 13, 3, 24, 23, 14]. It has been repeatedly pointed out that, though for known GLS the wave optics effects are small, they may have an observational interest in the near future. Recently Heyl [5, 6] turned attention that diffractive effects may be important in case of lensing by planetary masses and planetesimals. A number of authors (see, e.g., [14, 17] and references therein) pay attention to diffractive effects in GLS in connection with the future detection of gravitational waves.

The wave optics signals of the microlensed radiation from any real source are blurred because of its coherence properties; therefore they strongly depend upon the size of the extended source. The diffractive microlensing of an extended source has been studied in the case of caustic crossing events [7, 23], in the case of point mass lensing [14]; the mutual coherence of different lensed images of an extended source near the caustic has been estimated in a series of papers by Mandzhos [11, 12, 13]. In order to obtain a power spectrum of the microlensed radiation from an extended source, most research involves numerical calculations. Nevertheless, it would be desirable to have analytic results at least for some simple problems.

In the present paper we consider a Gaussian source under standard assumptions about correlation properties of incoherent source elements. We obtain a power spectrum of a radiation from the source, which is microlensed by one point mass. An observed radiation field is obtained in a standard way using the Kirhhoff integral; our new elements are related to the power spectrum. As distinct from earlier results by [14] on this problem, we propose an analytical representation of the power spectrum, which is convenient in case of a large source radius R as compared to the Einstein radius projected onto the source plane ($R_{E,s}$). In particular, we propose a closed-form relation in terms of the hypergeometric function for the power spectrum when the lensing mass is projected onto the centre of the source. Then we use this relation in order to study a general case of the source disposition and to

obtain approximations for the power spectrum for small $R_{E,s}/R$. We present the first orders of this expansion as well as an expansion in powers of $1/\omega^2$ in the high-frequency case.

2 Basic relations

In this section we formulate general suppositions and relations used below. Following [3, 14] and many other papers we use standard considerations of the diffraction theory and the gravitational lensing (see, e.g., [2, 15]); correspondingly we neglect polarization effects. Calculations of the field are performed in the flat space-time background; this is relevant in case of the Galactic systems. However the results can be easily extended to the case of extragalactic GLSs after a correct redefinition of cosmological distances.

Leaving aside the polarization effects we describe the radiation field by means of one scalar function $\varphi(t, \mathbf{r})$. One can assume that in the lens plane the light rays gain additional phase shifts corresponding to the gravitational time delays [2, 15]; after that the radiation field near the observer can be derived using the Kirchhoff's method.

Furthermore we work in the Cartesian coordinates; the observer, the lens and the source centre are situated in a neighborhood of Z-axis in the planes $z = 0$, $z = D_d$ and $z = D_s$ respectively. As we neglect polarization, we describe the source "current" by a scalar function $\mathrm{j}(t, \mathbf{r})$; for our considerations we can assume that this source lies completely in the source plane: $\mathbf{r} = (\mathbf{y}, D_s)$, $\mathbf{y} \in \mathbf{R}^2$. We also assume that $\mathrm{j}(t, \mathbf{r})$ is a stochastic process having the correlation properties

$$< \mathrm{j}(t, \mathbf{r})\mathrm{j}(t', \mathbf{r}') >= \delta(\mathbf{y} - \mathbf{y}')I(t - t', \mathbf{y}), \tag{1}$$

which take into account that the emission from different points of the source is incoherent; $< ... >$ represent an ensemble average.

For the Fourier transforms $\tilde{\mathrm{j}}(k, \mathbf{r}) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} \mathrm{j}(t, \mathbf{r})e^{i\omega t}dt$ we have then

$$< \tilde{\mathrm{j}}(\omega, \mathbf{r})\tilde{\mathrm{j}}^*(\omega', \mathbf{r}') >= \delta(\mathbf{y} - \mathbf{y}')\delta(\omega - \omega')\, \tilde{I}(\omega, \mathbf{y}), \tag{2}$$

where $\tilde{I}(\omega, \mathbf{y}) = \int dt\, I(t, \mathbf{y})e^{i\omega t}$ is an intensity of an element at point $\mathbf{y}$ for frequency ω.

The Helmholtz equation for the Fourier transformed radiation field $\tilde{\varphi}(\omega, \mathbf{r})$

$$\Delta\tilde{\varphi} + \omega^2\tilde{\varphi} = -4\pi\tilde{j} \quad (c = 1).$$

We consider solution $\tilde{\varphi}(\omega, \mathbf{r})$ of this equation describing the radiation from the source plane $\mathbf{r}' = (\mathbf{r}'_\perp, D_s)$, $\mathbf{r}'_\perp \in \mathbf{R}^2$. Let $D_{ds} = D_s - D_d$ be the distance from the lens plane to the source plane (in the flat space). Near Z-axis ($|\mathbf{r}'_\perp| << D_{ds}$) in the lens plane $\mathbf{r} = (\mathbf{r}_\perp, D_d)$ and under assumption that $R << D_{ds}$, where R is a source size, we have

$$\tilde{\varphi}(\omega, \mathbf{r}) = \frac{e^{i\omega D_{ds}}}{D_{ds}} \int d^2\mathbf{r}'_\perp \, \tilde{\mathrm{j}}(\omega, \mathbf{r}'_\perp) \exp\left[\frac{i\omega}{2D_{ds}} (\mathbf{r}_\perp - \mathbf{r}'_\perp)^2\right]. \tag{3}$$

In order to calculate the field after passing the lens plane we use the well known approximation (e.g., [2]) assuming that the radiation gains an additional phase shift ωt_{grav} after passing the lens plane $\mathbf{r} = (D_d, \mathbf{r}_\perp)$. Then the transfer function is introduced

$$T(\omega, \mathbf{y}) = \exp\{-i\omega t_{grav}(\mathbf{y})\},$$

where $t_{grav}(\mathbf{y})$ is the general relativistic time delay, which is the same for all frequencies. The solution just after passing the lens plane is

$$\tilde{\varphi}_s(\omega, \mathbf{y}') = \frac{e^{i\omega D_{ds}}}{D_{ds}} T(\omega, \mathbf{y}') \int d^2\mathbf{y} \, \tilde{\mathrm{j}}(\omega, \mathbf{y}) \exp\left[i\omega \frac{(\mathbf{y}' - \mathbf{y})^2}{2D_{ds}}\right].$$

The radiation comes to an observer at the origin $\mathbf{r} = 0$. The field $\tilde{\varphi}(\omega, \mathbf{0})$ is calculated using the Kirhhoff-Sommerfeld method using known field $\tilde{\varphi}_s(\omega, \mathbf{y}')$ for $\mathbf{r} = (\mathbf{y}', D_d)$:

$$\tilde{\varphi}(\omega, \mathbf{0}) = \frac{\omega e^{i\omega D_d}}{2\pi i D_d} \int d^2\mathbf{y}' \tilde{\varphi}_s(\omega, \mathbf{y}') \exp\left[\frac{i\omega}{2D_d} \mathbf{y}'^2\right]. \tag{4}$$

If the lens be absent, then instead of $\tilde{\varphi}_s$ in this formula we must use (3) yielding the same unperturbed solution near the observer.

The field near the observer is then

$$\tilde{\varphi}(\omega, \mathbf{0}) =$$

$$= \frac{\omega e^{i\omega D_s}}{2\pi i D_d D_{ds}} \int d^2\mathbf{y}' T(\omega, \mathbf{y}') \int d^2\mathbf{y} \, \tilde{\mathrm{j}}(\omega, \mathbf{y}) \exp\left\{i\omega \left[\frac{(\mathbf{y}' - \mathbf{y})^2}{2D_{ds}} + \frac{(\mathbf{y}')^2}{2D_d}\right]\right\} =$$

$$= \frac{e^{i\omega D_s}}{2i D_d D_{ds}} \int d^2\mathbf{y} \, \tilde{\mathrm{j}}(\omega, \mathbf{y}) \phi(\omega, \mathbf{y}) \tag{5}$$

where

$$\phi(\omega, \mathbf{y}) = \frac{\omega}{\pi} \int \exp\left\{ i\omega \left[\frac{(\mathbf{x}-\mathbf{y})^2}{2D_{ds}} + \frac{\mathbf{x}^2}{2D_d} - t_{grav}(\mathbf{x}) \right] \right\} d^2\mathbf{x} \,. \tag{6}$$

Using (2) we obtain

$$< \tilde{\varphi}(\omega, \mathbf{0}) \tilde{\varphi}^*(\omega', \mathbf{0}) >= \delta(\omega - \omega') P(\omega)$$

with the power spectrum

$$P(\omega) = \left(\frac{1}{2\, D_d D_{ds}} \right)^2 \int d^2\mathbf{y}\, \tilde{I}(\omega, \mathbf{y})\, |\phi(\omega, \mathbf{y})|^2 \,. \tag{7}$$

3 Central Gaussian source and point lensing mass

Relation (7) is written for arbitrary sources and gravitational time delays. Further we consider the case of one point microlens at $\mathbf{r} = (\mathbf{0}, D_d)$ with the delay time

$$t_{grav}(\mathbf{y}) = 2r_g \ln(|\mathbf{y}|/L), \quad r_g = 2Gm; \tag{8}$$

here m is the microlens mass, L is a dimensional parameter which disappears in final calculations; further it is omitted.

We assume the Gaussian brightness distribution over the source for the intensity $\tilde{I}$ from equation (2):

$$I(\omega, \mathbf{y}, \mathbf{r}_0) = \frac{f(\omega)}{\pi R^2} \exp\left(-\frac{(\mathbf{y} - \mathbf{r}_0)^2}{R^2} \right); \tag{9}$$

here $\mathbf{r}_0$ is the source centre in the source plane; function $f(\omega)$ is supposed to be the same for all source points; this function determines coherence properties of every emitting source element.

On account of equation (8) we have

$$\phi(\omega, \mathbf{y}) = \frac{\omega}{\pi} \int \exp\left\{ i\omega \left[\frac{(\mathbf{x}-\mathbf{y})^2}{2D_{ds}} + \frac{\mathbf{x}^2}{2D_d} - 2r_g \ln|\mathbf{x}|) \right] \right\} d^2\mathbf{x} \tag{10}$$

This integral can be calculated exactly [3, 2, 14] using the confluent hypergeometric function $\Phi(a, c; x)$ [1]:

$$\phi(\omega,\mathbf{y}) = \omega^{i\sigma} e^{\frac{i\omega \mathbf{y}^2}{2D_{ds}}}\Gamma(1-i\sigma)\left(\frac{2D_{ds}D_d i}{D_s}\right)^{1-i\sigma}\Phi\left(i\sigma,\ 1;\ -\frac{i\omega D_d y^2}{2D_{ds}D_s}\right) =$$

$$= \omega^{i\sigma} e^{\frac{i\omega \mathbf{y}^2}{2D_s}}\Gamma(1-i\sigma)\left(\frac{2D_{ds}D_d i}{D_s}\right)^{1-i\sigma}\Phi\left(\ i\sigma, 1;\ i\sigma y^2/R_{E,s}^2\right); \qquad (11)$$

where we used the Kummer transformation [1]; $\sigma = \omega r_g$, $R_{E,s} = [2r_g D^*]^{1/2}$, is the Einstein radius projected onto the source plane, $D^* = D_{ds}D_s/D_d$; $y = |\mathbf{y}|$.

For the Gaussian source the power spectrum (7) has the form

$$P(\omega,\mathbf{r}_0) = \left(\frac{1}{2\,D_d D_{ds}}\right)^2 \frac{f(\omega)}{\pi R^2}\int d^2\mathbf{y}\ \exp\left[-\frac{(\mathbf{y}-\mathbf{r}_0)^2}{R^2}\right]|\phi(\omega,\mathbf{y})|^2 \qquad (12)$$

The microlensing effect can be described by the ratio

$$\Upsilon = P(\omega,\mathbf{r}_0)/P_0(\omega), \qquad (13)$$

where P_0 is the power spectrum in the absence of microlensing ($\sigma = 0$).

If the source centre is at the origin ($\mathbf{r}_0 = 0$), the integral (12) can be estimated in terms of hypergeometric functions [19]; the derivation (that can be found in Appendix A) yields $\Upsilon = \Upsilon_0(\alpha,\sigma)$ where

$$\Upsilon_0(\alpha,\sigma) \equiv \exp[2\sigma\arctan(\beta)]\,|\Gamma(1-i\sigma)|^2\,F\left(i\sigma,-i\sigma;\ 1\,;(1+\beta^2)^{-1}\right); \quad (14)$$

$\beta = \alpha/\sigma$, $\alpha = R_{E,s}^2/R^2$, $F(a,b;c;x)$ is the hypergeometric function. Note that $\sqrt{\beta} = R^{-1}\sqrt{\lambda D^*/\pi}$ is essentially a ratio of the size of the Fresnel zone to the source size.

For large $\alpha >> 1$ and $\sigma \sim O(1)$ the argument of the hypergeometric function is small and we have $\Upsilon_0(\alpha,\sigma) \approx 2\pi\sigma$.

For $\alpha << 1$ that is $R_{E,s} << R$, and bounded $\sigma \sim O(1)$ it is convenient to use an expansion of the hypergeometric function $F(a,b;c;x)$ near the point $x = 1$ for an integer c. Then (14) takes on the form

$$\Upsilon_0(\alpha,\sigma) =$$

$$= \exp[2\sigma\arctan(\beta)]\left\{1-\sigma^2 s\sum_{n=0}^{\infty}\left|\frac{\Gamma(n+1+i\sigma)}{\Gamma(1+i\sigma)}\right|^2\frac{s^n[k_n(\sigma)-\ln s]}{(n+1)(n!)^2}\right\}, \quad (15)$$

where

$$k_n(\sigma) = 2\psi(n+1) - \psi(n+1+i\sigma) - \psi(n+1-i\sigma) + \frac{1}{n+1}, \tag{16}$$

$s = \alpha^2/(\alpha^2+\sigma^2)$, $\quad \psi(x) \equiv d\ln\Gamma(x)/dx$.
We shall write the expansion in α up to terms $\sim \alpha^2$ and $\alpha^2 \ln\alpha$ that preserve a dependence upon the frequency. In this approximation equation (15) can be written as

$$\Upsilon_0(\alpha,\sigma) = 1 + 2\alpha + 2\alpha^2 - \alpha^2[k_0 - 2\ln(\alpha/\sigma)] \quad . \tag{17}$$

The expansion (15) can be rewritten in the form which is convenient to look for asymptotic expansions at large frequencies:

$$\Upsilon_0(\alpha,\sigma) = \exp[2\sigma\arctan(\alpha/\sigma)] \left\{1 - \sum_{n=0}^{\infty} C_n(\sigma) \frac{\tilde{s}^{n+1}[\tilde{k}_n(\sigma) - \ln\tilde{s}]}{(n+1)(n!)^2}\right\}, \tag{18}$$

where $\tilde{s} = \sigma^2 s = \alpha^2/(1+\beta^2)$,

$$\tilde{k}_n(\sigma) = k_n(\sigma) + 2ln(\sigma), \quad C_n(\sigma) = \left|\frac{\Gamma(n+1+i\sigma)}{\Gamma(1+i\sigma)\sigma^n}\right|^2 = \prod_{m=0}^{n}\left(1 + \frac{m^2}{\sigma^2}\right), \tag{19}$$

Using asymptotic relations for function ψ (e.g., [1]) for large arguments, the coefficients $\tilde{k}_n(\sigma)$ can be expanded in powers of σ^{-2}. Therefore, one can write

$$\Upsilon_0(\alpha,\sigma) = \sum_{n=0}^{m} \sigma^{-2n}\Upsilon_0^{(n)}(\alpha) + O(\sigma^{-2(m+1)}), \tag{20}$$

Up to the terms $\sim \sigma^{-2}$ we have

$$\tilde{k}_n(\sigma) = \tilde{k}_n(\infty) - \frac{1}{\sigma^2}\left[n(n+1) + \frac{1}{6}\right] + O(\sigma^{-4}), \tag{21}$$

where $\tilde{k}_n(\infty) = 2\psi(n+1) + (n+1)^{-1}$; from (19) it follows that

$$C_n(\sigma) = 1 + \frac{n(n+1)(2n+1)}{6\,\sigma^2} + ...$$

Then we have the geometric optics limit ($\sigma \equiv \omega r_g \to \infty$)

$\Upsilon_0(\alpha,\infty) = \Upsilon_0^{(0)}(\alpha) =$

$$= \exp(2\alpha)] \left\{ 1 - \alpha^2 \sum_{n=0}^{\infty} \frac{\alpha^{2n}[\tilde{k}_n(\infty) - 2\ln\alpha]}{(n+1)(n!)^2} \right\} = 2\alpha e^{2\alpha} K_1(2\alpha), \qquad (22)$$

where we used a representation for modified Bessel function K_1 (see, e.g., [4]; formula 8.446). A direct calculation of the amplification coefficient from the geometric optics for the point mass lens (Appendix B) is in accordance with this expression.

For the next term of the expansion we have

$$\Upsilon_0^{(1)}(\alpha) = -\alpha^2 e^{2\alpha} \left\{ \frac{2}{3}\alpha + \sum_{n=0}^{\infty} \frac{C_n^{(1)}(\alpha)\alpha^{2n}}{(n+1)(n!)^2} \right\},$$

$$C_n^{(1)}(\alpha) = \left[\tilde{k}_n(\infty) - 2\ln\alpha\right] \left[\frac{n}{6}(n+1)(2n+1) - \alpha^2(n+1)\right] + \alpha^2 - n(n+1) - \frac{1}{6}.$$

4 Non-central source

Now we proceed to derivation of Υ in case of a non-central source using equations (14)-(17). Integration over the angular variable yields

$$P(\omega, \mathbf{r}_0) = (2D_d D_{ds})^{-2}\, 2f(\omega) R^{-2} \cdot$$

$$\cdot \exp\left(-\frac{\mathbf{r}_0^2}{R^2}\right) \int_0^\infty dy\, y \exp\left(-\frac{y^2}{R^2}\right) I_0\left(\frac{2yr_0}{R^2}\right) |\phi(\omega,\mathbf{y})|^2, \qquad (23)$$

where $r_0 = |\mathbf{r}_0|$. Using expansion of modified Bessel function of the 1-st kind I_0 in the form of Taylor series and the substitution $y^2 = tR_{E,s}^2$ we get

$$\Upsilon(\alpha,\sigma,r_0) = \alpha \exp\left(-\frac{r_0^2}{R^2} + \pi\sigma\right) |\Gamma(1-i\sigma)|^2 \cdot$$

$$\cdot \sum_{n=0}^{\infty} \frac{\alpha^n}{(n!)^2} \left(\frac{r_0}{R}\right)^{2n} \int_0^\infty dt\, e^{-\alpha t} t^n \left|\Phi\left(\, i\sigma, 1;\; i\sigma t\right)\right|^2. \qquad (24)$$

Using differentiation with respect to parameter α and taking into account equations (11),(14) this can be represented as

$$\Upsilon(\alpha,\sigma,r_0) = \exp\left(-\frac{r_0^2}{R^2}\right)\sum_{n=0}^{\infty}(-1)^n\frac{\alpha^{n+1}}{(n!)^2}\left(\frac{r_0}{R}\right)^{2n}\frac{\partial^n}{\partial\alpha^n}\left[\frac{\Upsilon_0(\alpha,\sigma)}{\alpha}\right]. \qquad (25)$$

Obviously this representation is workable when r_0/R is not large; this will be supposed further. This excludes, e.g., the case of a point source.

Simple analysis shows that, to obtain the inputs up to the orders of α^2 and $\alpha^2\ln\alpha$ into (25), it is sufficient to use the truncated expression (17). We thus get

$$\Upsilon(\alpha,\sigma,r_0) = 1 + 2\alpha e^{-r_0^2/R^2} + \alpha^2 e^{-r_0^2/R^2}\cdot$$

$$\cdot\left\{\left(1-\frac{r_0^2}{R^2}\right)[2-k_0(\sigma)-2\ln(\sigma/\alpha)] - 2\frac{r_0^2}{R^2} + 2g\left(\frac{r_0^2}{R^2}\right)\right\} \qquad (26)$$

where

$$g(x) = \sum_{n=2}^{\infty}\frac{x^n}{n(n-1)n!} = (x-1)[Ei(x)-C-\ln x] - e^x + 1 + 2x,$$

$Ei(x)$ is the integral exponent, C is the Euler constant.

Now we proceed to write an asymptotic expansion for $\Upsilon(\alpha,\sigma,r_0)$ for large frequencies. Substitution of (20) into (25) leads to an asymptotic series in powers of σ^{-2}:

$$\Upsilon(\alpha,\sigma,r_0) = \sum_{m}^{M}\sigma^{-2m}\Upsilon^{(m)}(\alpha,r_0) + O(\sigma^{-2(M+1)}) \qquad (27)$$

with the coefficients

$$\Upsilon^{(m)}(\alpha,r_0) = \exp\left(-\frac{r_0^2}{R^2}\right)\sum_{n=0}^{\infty}(-1)^n\frac{\alpha^{n+1}}{(n!)^2}\left(\frac{r_0}{R}\right)^{2n}\frac{\partial^n}{\partial\alpha^n}\left[\frac{\Upsilon_0^{(m)}(\alpha)}{\alpha}\right]. \qquad (28)$$

Some caution is needed when we differentiate term by term asymptotic expansion (20); however, this can be easily justified in view of the explicit structure of the functions involved. The geometric optics limit yields

$$\Upsilon(\alpha,\infty,r_0) = \Upsilon^{(0)}(\alpha,r_0), \qquad (29)$$

where $\Upsilon^{(0)}(\alpha, r_0)$ is given by (28) for $m = 0$. A direct calculation of the amplification using standard gravitational lensing theory yields the same result (see Appendix B).

The coefficient $\Upsilon^{(1)}(\alpha)$, which describes the first correction to (29) with an accuracy up to $\sim \alpha^2$, can be obtained either from (28) or from (26):

$$\Upsilon^{(1)}(\alpha) = \frac{1}{6}\alpha^2 \exp\left(-\frac{r_0^2}{R^2}\right)\left(1 - \frac{r_0^2}{R^2}\right). \tag{30}$$

5 Discussion

We have obtained analytical expressions for the power spectrum of the radiation of an extended Gaussian source microlensed by a point mass with standard assumptions concerning the incoherent emission of the source elements. If the source centre, the lensing mass and the observer are on a straight line, the power spectrum is given by equation (14) in terms of the hypergeometric function. In the case of the general arrangement the result is presented in the form of functional series (25). This representation of the power spectrum makes it possible to derive approximations up to any accuracy in case of a sufficiently small $\alpha = (R_{E,s}/R)^2$. The representation is effective under condition that the source size is not small as compared to the distance to the source centre. The opposite case can be treated by the method of [14], where an expansion of the magnification around the source position is used. Representations (14,25) are used to derive lowest orders of the asymptotic expressions in cases of the small lens (26) and the high-frequencies (30). We have shown by means of a direct calculation that the high frequency limit yields exactly the expressions of the geometric optics. As we see from (17,30), the first nontrivial wave optics contribution, which is dependent upon the frequency, appears in the terms $\sim \alpha^2$ (though the first geometric optics lensing contribution has the order $\sim \alpha$).

At the end of the paper we give some estimates for the radio waveband; though much smaller wavelengthes also may be of interest (cf., e.g., [16]). For the effects of the wave optics to be significant, one must have $\sigma = r_g\omega \sim 1$ and $\alpha \sim 1$. For a wavelength $\sim 1 - 10\,cm$ the first condition is fulfilled if the microlens mass is of the order of $10^{-5}M_\odot$. Such planetary mass objects must be common in the Milky Way, and there is a number of observational confirmations of exosolar planets, including observations by means of the

microlensing (see, e.g., [20]). In order that α not be small, the source size R must be of the order or less than $R_{E,s}$. This is true for a wide interval of D^* in the case of the Milky Way objects; though the probability of microlensing of a suitable radio source is small (see, however, [6]). For a distant extragalactic source at $D_s \approx D_{ds} \sim 10^3 Mpc$ microlensed by a planet at $D_d = 10\,kpc$, then $R_{E,s} = 10^{-2}[M/(10^{-5}M_{\odot})]^{1/2}pc$ must be of the order of an inhomogeneity scale in the radio waveband. The probability of this microlensing can be compared to that of microlensing of quasars by Milky Way stars and stellar remnants, and it was pointed out [21] that observation of such events could be observationally relevant in the near future.

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Appendix A. **Derivation of equation (14) in the case of the central source.**

From equation (10) we have

$$|\phi(\omega, \mathbf{y})|^2 = |\Gamma(1 - i\sigma)|^2 \left(\frac{2D_{ds}D_d}{D_s}\right)^2 e^{\sigma\pi} \left|\Phi\left(\, i\sigma, 1;\ i\sigma y^2/R_{E,s}^2\right)\right|^2. \tag{31}$$

For $r_0 = 0$ the power spectrum (12) equals to

$$P(\omega, 0) = \left(\frac{1}{2\,D_d D_{ds}}\right)^2 \frac{2f(\omega)}{R^2} \int_0^\infty dy\, y \exp\left(-\frac{y^2}{R^2}\right) |\phi(\omega, y)|^2, \tag{32}$$

where an integration over the angular variable is performed. On account of (31,32) and using the substitution $t = y^2$, we obtain the ratio (13) corresponding to $\mathbf{r}_0 = 0$ as follows

$$\Upsilon_0(\alpha,\sigma) = e^{\sigma\pi}\left|\Gamma(1-i\sigma)\right|^2 \int dt\, e^{-t} \left|\Phi\left(i\sigma, 1;\; i\sigma t/\alpha\right)\right|^2 \;\;, \qquad (33)$$

where $\alpha = R_{E,i}^2/R^2$.

The integral in formula (33) is a special case of an expression that could be obtained by means of formula 6.15.22 of book ([1]). Here, in order to estimate (33), we shall perform calculations directly for the integral

$$I(a,\lambda,a',\lambda') = \int\limits_0^\infty dt\, e^{-t} \Phi\left(a, 1;\; \lambda t\right) \Phi\left(a', 1;\; \lambda' t\right), \qquad (34)$$

where $0 < \mathrm{Re}(a) < 1$, $0 < \mathrm{Re}(a') < 1$, $|\lambda'| + |\lambda| < 1$. With this aim we need the well-known representations for the hypergeometric function

$$F\left(a, b; 1;\; x\right) = \frac{1}{\Gamma(b)\Gamma(1-b)} \int\limits_0^1 du\, \left(1 - zt\right)^{-a} t^{b-1} (1-t)^{-b} =$$

$$= \frac{1}{B(b, 1-b)} \int\limits_0^\infty d\tau\; \left[1 + (1-z)\,\tau\right]^{-a} \tau^{b-1} (1+\tau)^{a-1} \quad , \qquad (35)$$

where we used the Beta-function $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ and the substitution $t = \tau/(1+\tau)$, and for the confluent hypergeometric function

$$\Phi\left(a, 1;\; x\right) = \frac{1}{B(a, 1-a)} \int\limits_0^1 du\; e^{xu} u^{a-1} (1-u)^{-a} \quad . \qquad (36)$$

We use (36) for $\Phi\left(a, 1;\; \lambda t\right)$ and $\Phi\left(a', 1;\; \lambda' t\right)$ in (34) and we perform integration over t after change of the integration order:

$$I(a,\lambda,a',\lambda') =$$

$$= \frac{1}{B(a,1-a)B(a',1-a')} \int\limits_0^1 du\; u^{a-1}(1-u)^{-a} \int\limits_0^1 dv\; \frac{v^{a'-1}(1-v)^{-a'}}{1 - \lambda u - \lambda' v}$$

Integration over dv, after substitution $v \to \xi$: $v = \tau/(1+\tau)$ and $\tau = \xi(1-\lambda u)/(1-\lambda-\lambda u)$, is easily carried out using an integral representation for the Beta-function.

Then substitution $u = \eta/(1-\lambda+\eta)$ yields

$$I(a,\lambda,a',\lambda') = \frac{(1-\lambda)^{-a}(1-\lambda')^{-a'}}{B(a,1-a)} \int\limits_0^\infty d\eta \, \frac{\eta^{a-1}(1+\eta)^{a'-1}}{[1+(1-z)\eta]^{a'}}$$

where $z = \lambda\lambda'(1-\lambda)^{-1}(1-\lambda')^{-1}$. Therefore, on account of the representation (35) we get a final relation

$$I(a,\lambda,a',\lambda') = \frac{F(a',a;\ 1;\ z)}{(1-\lambda)^a(1-\lambda')^{a'}}. \tag{37}$$

All calculations are fulfilled in the domain of the parameters a, λ, a', λ' where the integrals involved are convergent. However relation (37) can be analytically continued to a more wide domain which includes the values $a = i\sigma,\ a' = -i\sigma,\ \lambda = i\sigma/\alpha,\ \lambda' = -i\sigma/\alpha$. This yields equation (14) of the main text.

Appendix B. **Gaussian source amplification in geometric optics.**

The amplification of a point source by a point mass lens within the geometric optics in variables of the source plane is well known [15, 2]:

$$\mu(y) = \frac{y^2 + 2R_{E,s}^2}{y\sqrt{y^2 + 4R_{E,s}^2}} \tag{38}$$

In our case this must be convolved with the Gaussian brightness distribution (9) yielding the total amplification

$$A = \frac{1}{\pi R^2}\int d^2\mathbf{y}\mu(y)e^{-(\mathbf{y}-\mathbf{r_0})^2/R^2} = \frac{2}{R^2}e^{-\frac{r_0^2}{R^2}}\int\limits_0^\infty dy\, y\, \mu(y) e^{-\frac{y^2}{R^2}} I_0\left(\frac{2yr_0}{R^2}.\right)$$

Taking into account (38), using the Taylor expansion of I_0 and substitution $y^2 = tR_{E,s}^2$ we have

$$A = A(\alpha, r_0) =$$

$$= e^{-\frac{r_0^2}{R^2}} \sum_{n=0}^{\infty} \frac{\alpha^{n+1}}{(n!)^2}\left(\frac{r_0}{R}\right)^{2n} \int\limits_0^\infty dy\, e^{-\alpha t} t^n \frac{t+2}{\sqrt{t^2+4t}} =$$

$$= e^{-\frac{r_0^2}{R^2}} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{n+1}}{(n!)^2} \left(\frac{r_0}{R}\right)^{2n} \frac{\partial^n}{\partial \alpha^n} \int\limits_0^{\infty} dy\, e^{-\alpha t} \frac{t+2}{\sqrt{t^2+4t}} \tag{39}$$

The latter integral can be written [4] in terms of modified Bessel function of the second kind K_1 yielding

$$A(\alpha, 0) = \alpha \int\limits_0^{\infty} dt\, e^{-\alpha t} \frac{t+2}{\sqrt{t^2+4t}} = 2\alpha e^{2\alpha} K_1(2\alpha) \tag{40}$$

This is the same as $\Upsilon_0(\alpha, \infty)$ of (22).

For an arbitrary source position from (39) we have

$$A(\alpha, r_0) = e^{-\frac{r_0^2}{R^2}} \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{n+1}}{(n!)^2} \left(\frac{r_0}{R}\right)^{2n} \frac{\partial^n}{\partial \alpha^n} \left[\frac{A(\alpha, 0)}{\alpha}\right], \tag{41}$$

which is the same as $\Upsilon(\alpha, \infty, r_0)$ of (29).